



PERTH MODERN SCHOOL

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INDEPENDENT PUBLIC SCHOOL

Semester Two Examination, 2020

Question/Answer booklet

## MATHEMATICS SPECIALIST

### UNIT 1&2

Section Two:

Calculator-assumed

Your Name \_\_\_\_\_

Your Teacher's Name \_\_\_\_\_

#### Time allowed for this section

Reading time before commencing work: ten minutes  
Working time: one hundred minutes

#### Materials required/recommended for this section

##### *To be provided by the supervisor*

This Question/Answer booklet  
Formula sheet (retained from Section One)

##### *To be provided by the candidate*

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

#### Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question	Marks	Max	Question	Marks	Max
9		10	16		6
10		6	17		6
11		5	18		8
12		9	19		8
13		7	20		8
14		10			
15		10			

## Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	50	35
Section Two: Calculator-assumed	12	12	100	93	65
<b>Total</b>					100

## Instructions to candidates

- The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2019*. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer booklet.
- You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
- Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
- Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you **do not use pencil**, except in diagrams.

**See Next Page**

**Section Two: Calculator-assumed****(93 Marks)**

This section has **12 (twelve)** questions. Answer **all** questions. Write your answers in the spaces provided.

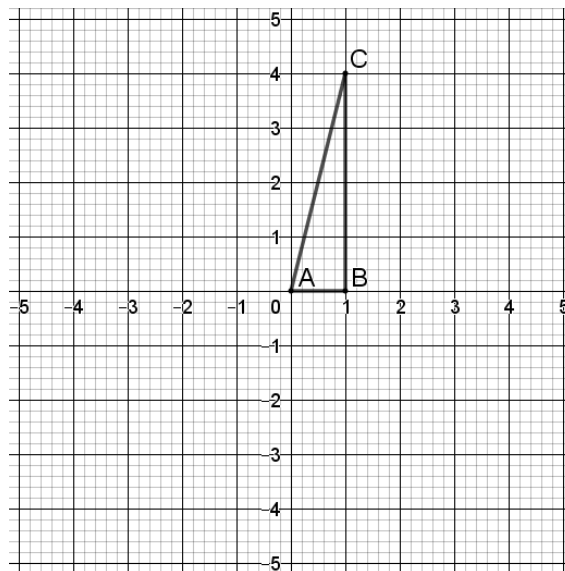
Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 100 minutes.

**Question 9 (2.2.1, 2.2.5-2.2.7, 2.2.9, 2.2.10)****(10 marks)**

Consider the triangle with vertices  $A(0,0)$ ,  $B(1,0)$  and  $C(1,4)$ , plotted below.



- a) The triangle is transformed by a matrix  $M$  to give an image with vertices  $A'(0,0)$ ,  $B'(0,1)$  and  $C'(-4,1)$ . Write down the matrix  $M$ . (2 marks)

Question 9 continued

- b) Triangle  $A'B'C'$  (the **image** from part (a) ) is transformed by a matrix  $N$  to give an image with vertices  $A''(0,0)$ ,  $B''(0,-1)$  and  $C''(-4,-1)$ . Write down the matrix  $N$ . (2 marks)
- c) **Hence** write down the matrix  $P$  which would transform triangle  $ABC$  to triangle  $A''B''C''$ , showing your working. (3 marks)
- d) The triangle  $ABC$  is transformed by a matrix  $Q$  to a triangle with coordinates  $A'''(0,0)$ ,  $B'''(2,1)$  and  $C'''(2,7)$ . State the value of  $\det Q$ , given that  $\det Q > 0$ , justifying your answer. (3 marks)

**Question 10 (1.1.1, 1.1.2, 1.1.3, 1.1.4)****(6 marks)**

The genetic code is a set of rules defined by the four nucleotides of DNA, represented by the letters A, T, C and G. Three-letter nucleotide sequences are made from the four nucleotides.

- a) With no restrictions, how many 3-letter nucleotide sequences are possible in DNA?

(1 mark)

- b) How many 3-letter nucleotide sequences start with A and end with C?

(2 marks)

- c) How many 3-letter nucleotide sequences have a G at least twice?

(3 marks)

**Question 11 (2.3.7-2.3.11, 2.3.13-2.3.16)**

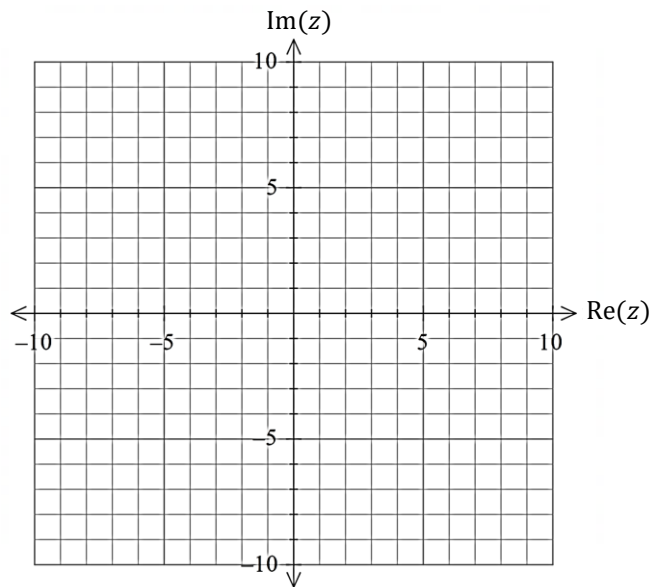
**(5 marks)**

Consider the following quadratic equation where  $c$  is a real number.

$$x^2 + 12x + c = 0$$

One of the solutions to this equation is  $z = -6 + 3i$ .

- a) Write down the other solution  $w$  of the equation, and plot (and label) both solutions in the complex plane below. (3 marks)

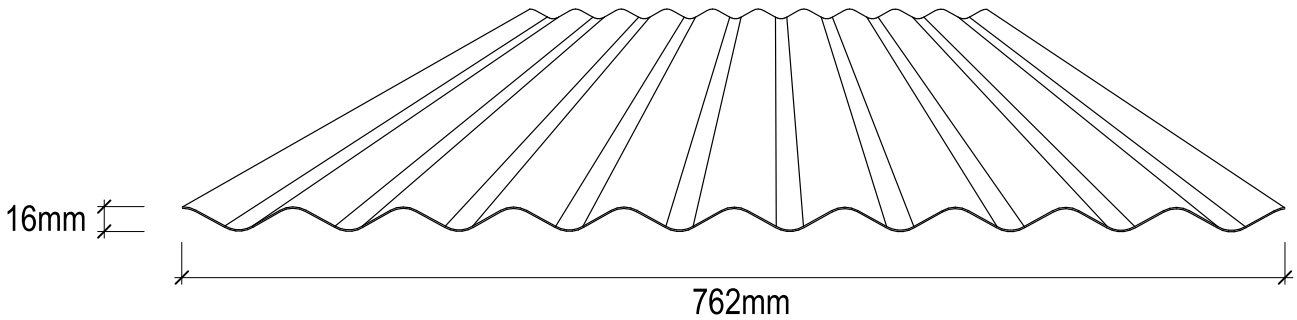


- b) Hence (or otherwise) determine the value of  $c$ . (2 marks)

**Question 12 (2.1.1-2.1.2, 2.1.9)**

**(9 marks)**

A roofing panel with the dimensions shown below has been left on the ground. An ant is walking across the top of the panel from the left end to the right end.



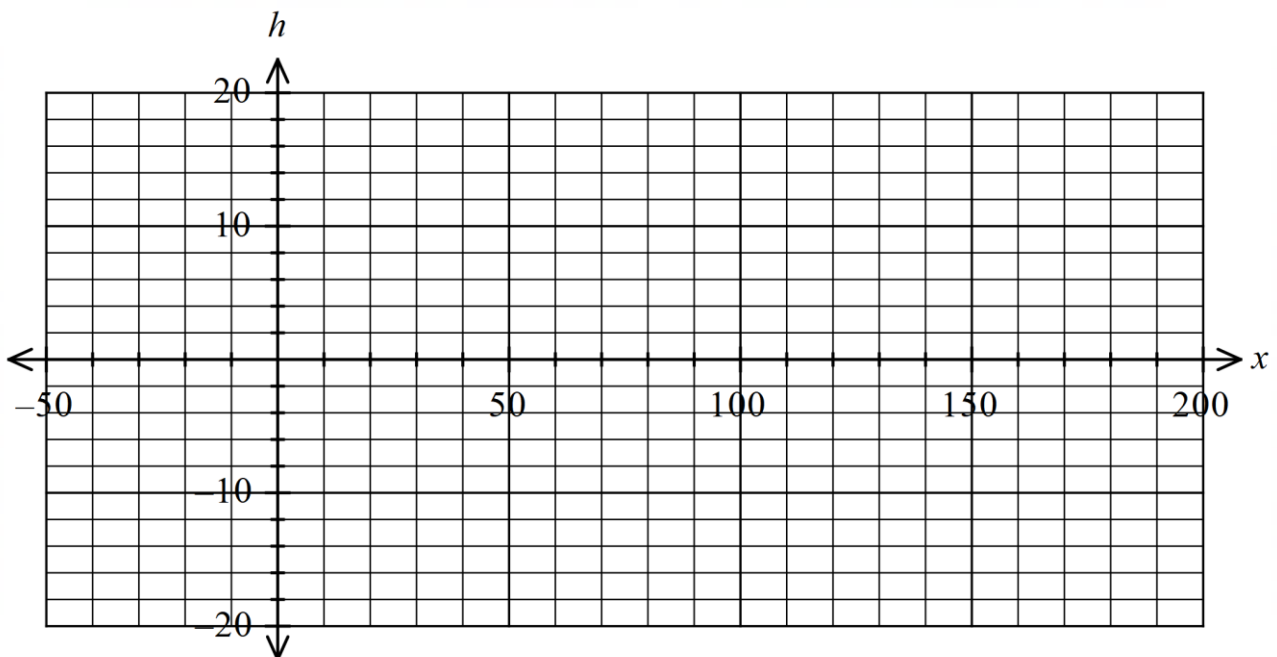
a) Write a function in the form

$$h = a \cos(b(x - c)) + d$$

modelling the height  $h$  mm that the ant is above the ground in terms of the horizontal distance  $x$  mm that the ant is from the left end of the panel. Specify the domain of the function. (Assume the panel has negligible thickness.) (3 marks)

b) Graph the function on the axes below.

(3 marks)



Question 12 continued

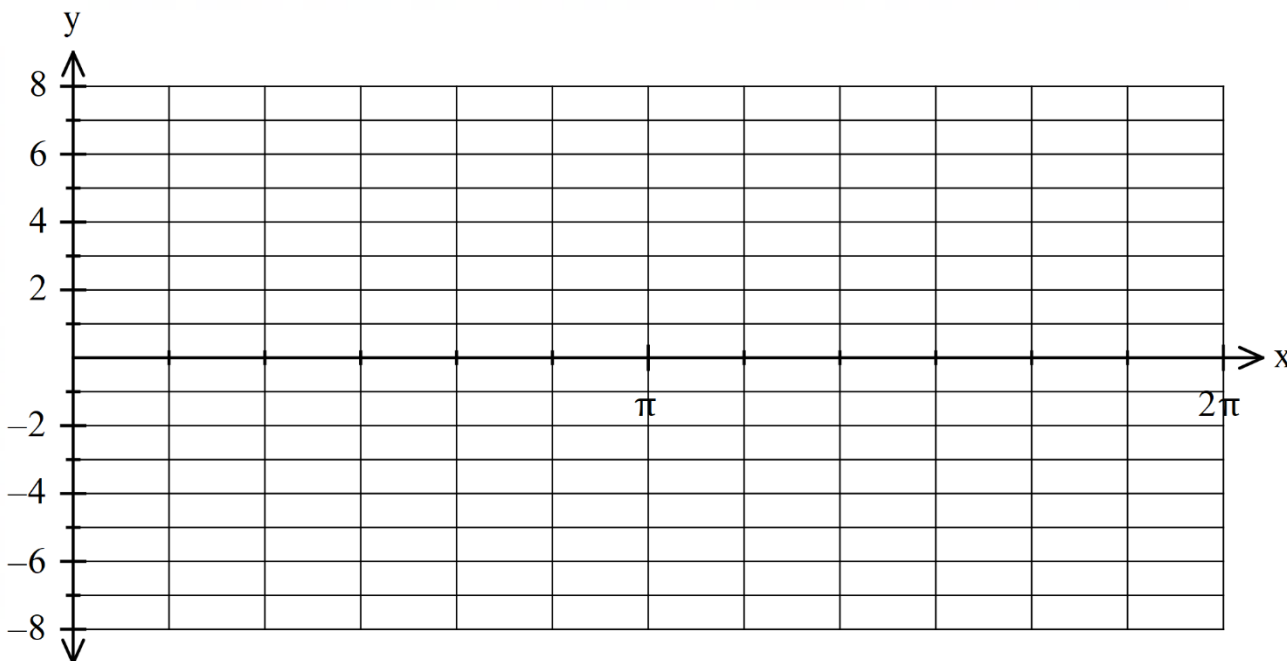
- c) The ant gets tired and stops to rest the third time he is climbing upwards at a height of 12 mm. How far (horizontally) does he have left to walk? (3 marks)

Question 13 (2.1.4)

(7 marks)

- a) Describe the transformation of  $y = \operatorname{cosec}(x)$  to  $y = 2 \operatorname{cosec}(x) - 4$ . (2 marks)

- b) Sketch  $y = 2 \operatorname{cosec}(x) - 4$  on the grid below, labelling all key features. (5 marks)





**Question 14 (1.2.6-1.2.13)****(10 marks)**

a) Three vectors are given by  $\mathbf{a} = 5\mathbf{i} - 12\mathbf{j}$ ,  $\mathbf{b} = -15\mathbf{i} + 10\mathbf{j}$  and  $\mathbf{c} = -7\mathbf{i} + y\mathbf{j}$  where  $y$  is a constant.

i) Determine the vector projection of  $\mathbf{b}$  on  $\mathbf{a}$  (give components as exact values).  
(3 marks)

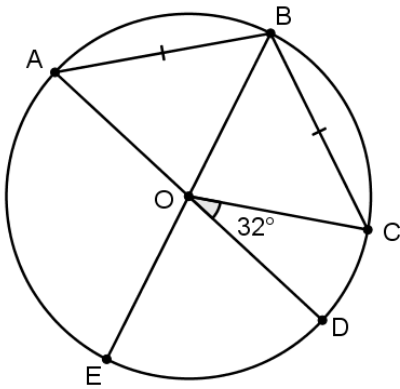
ii) Find  $y$  if the angle between  $\mathbf{b}$  and  $\mathbf{c}$  is  $45^\circ$ .  
(3 marks)

b) Vectors  $a\mathbf{i} + (a - 3)\mathbf{j}$  and  $(a - 7)\mathbf{i} + 5\mathbf{j}$  are perpendicular. Find the value(s) of  $a$  and the corresponding pairs of vectors.  
(4 marks)

**Question 15 (1.3.6-1.3.15)**

**(10 marks)**

- a) Consider the diagram below.  $\overline{AD}$  and  $\overline{BE}$  are diameters of the circle with centre  $O$ ,  $C$  lies on the circumference of the circle such that  $AB = BC$ , and  $\angle COD = 32^\circ$ .



Determine the sizes of the following angles

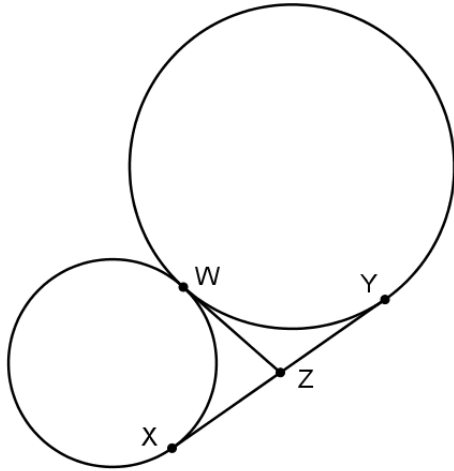
i)  $\angle AOB$

(2 marks)

ii)  $\angle CAE$

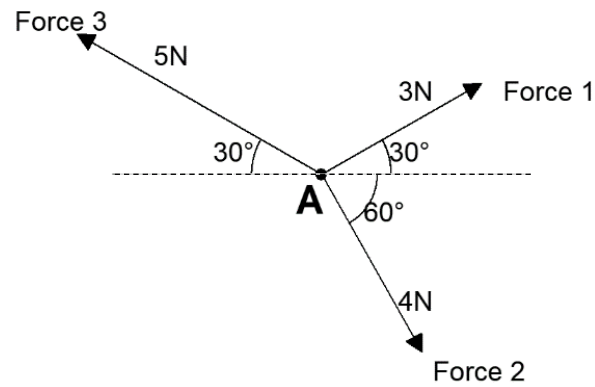
(3 marks)

- b) In the diagram below,  $W$  is the single point of intersection of the two circles. The segment  $\overline{XY}$  is tangent to both circles, intersecting with the circles at  $X$  and  $Y$ . Segment  $\overline{WZ}$  is also tangent to both circles, intersecting with  $\overline{XY}$  at  $Z$ . Prove that  $\triangle XWY$  is a right triangle. (5 marks)



**Question 16 (1.2.2, 1.2.8, 1.2.14)****(6 marks)**

Three forces act on the point  $A$  as shown. What is the magnitude of the resultant force acting on  $A$ , and in what direction would  $A$  move under these three forces? Give your answers to 2 decimal places, with the direction as an angle measured anticlockwise from the right (like the  $30^\circ$  angle for Force 1).



**Question 17 (2.2.1, 2.2.2)****(6 marks)**

- a) Given invertible  $n \times n$  matrices  $A$ ,  $B$ ,  $C$  and  $X$  with  $AX - B = CBX$ , write  $X$  in terms of  $A$ ,  $B$  and  $C$ .

(3 marks)

- b) Solve the following matrix equation for  $Y$  (given that  $Y$  is a  $2 \times 2$  matrix).

$$3Y - Y \begin{bmatrix} 1 & -3 \\ 1 & 6 \end{bmatrix} = 5I$$

(3 marks)

**Question 18 (1.1.7, 1.1.8)****(8 marks)**

Four Year 10 students and eleven Year 11 students from Western Australia are nominated as candidates for a Mathematics Summer Camp. How many ways can a group of four participants be selected:

a) without restriction? (2 marks)

b) if the only student from Bunbury must be included? (2 marks)

c) if there must be exactly two Year 11 students? (2 marks)

d) if there must be at least one Year 10 student? (2 marks)

**Question 19 (1.1.5, 1.1.9)****(8 marks)**

a) How many integers between 1 and 101 are multiples of 5, 6 or 7?

(4 marks)

b) Use the fact that  ${}^nC_r = \frac{n!}{(n-r)!r!}$  to show that  ${}^{n-1}C_{r-1} \times n = {}^nC_r \times r$ . (4 marks)

**Question 20 (2.2.1-2.2.10)****(8 marks)**

$$\text{Let } A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}.$$

a) Calculate  $A^2$  (that is,  $A \times A$ ). Show working and simplify your answer. (3 marks)

b) Calculate the product  $A^3$  by multiplying your answer to part (a) by  $A$  (you do not need to simplify your answer). (2 marks)

c) Determine a value of  $\alpha$  (with  $0 < \alpha < 2\pi$ ) such that  $A^3 = I$ . Justify your answer by referring to the linear transformation corresponding to the matrix  $A$ . (3 marks)

**END OF QUESTIONS**



**Additional working space**

Question number: \_\_\_\_\_

**Additional working space**

Question number: \_\_\_\_\_

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